

《概率论与数理统计》同步习题册参考答案

第一章 1.1 节

1. (1) $\{x | 0 \leq x \leq 100\}$; (2) $\{(x, y) | 0 \leq x^2 + y^2 \leq 1\}$; (3) $\{1, 2, 3, \dots\}$.
2. (1) $\overline{ABC}$; (2) ABC ; (3) $\overline{ABC} + \overline{ABC} + \overline{ABC}$; (4) $A \cup B \cup C$;
(5) $\overline{ABC} + \overline{ABC} + \overline{ABC} + \overline{ABC}$; (6) $\Omega - ABC$.
3. (1) (3) (4) (5) 成立.

1.2 节

1. 0.1. 2. $\frac{5}{8}$. 3. $\frac{1}{2}, \frac{1}{6}, \frac{3}{8}$. 4. 0.2. 5. 0.7.

1.3 节

1. $\frac{3!2!2!}{13!}$. 4. $\frac{6}{16}, \frac{9}{16}, \frac{1}{16}$. 3. $\frac{13}{21}$. 2. $\frac{7}{40}, \frac{3}{4}$. 5. $\frac{3}{4}$.

1.4 节

1. $1/3, 1/4$. 2. $\frac{1}{6}$. 3. $\frac{209}{300}, \frac{64}{209}$. 4. $\frac{19}{30}, \frac{48}{95}$.

1.5 节

1. 0.48. 2. $1 - 0.9 \times 0.95 \times 0.8$. 3. 0.896. 4. $\frac{4}{7}, \frac{3}{7}$.

第一章 自测题

- 一. 1. $\frac{2}{5}$. 2. $0, 1 - (p + q)$. 3. $\frac{2}{3}, \frac{1}{2}$. 4. $\frac{1}{3}$; 5. $\frac{2}{3}$. 6. 4.

7. $\frac{11}{27}$. 8. $\frac{2}{5}$. 9. 0.8. 10. 0.94. 11. $\frac{11}{30}$.

- 二. 1. A. 2. C. 3. B. 3. A. 4. A. 5. A.

- 三. 1. $1 - \frac{11^6}{12^6}, \frac{C_6^4 \cdot 11^2}{12^6}, \frac{C_6^4 \cdot 12 \cdot 11^2}{12^6}$. 2. $\frac{3}{5}, \frac{3}{4}, \frac{3}{10}, \frac{11}{27}, \frac{3}{5}$.

3. $\frac{40}{49}$. 4. $1 - 0.04^n > 0.999$. 5. 0.253, 47/253. 6. 1/4.

7. 0.24, 0.424.

第二章 2.1 节

1. $\frac{1}{2(2^{100} - 1)}, \frac{1}{3}$. 2. $P(X = 2) = \frac{1}{10}, P(X = 3) = \frac{9}{10}$.

3. $P(X = k) = \frac{5A_3^k}{8!}, k = 0, 1, 2, 3$. 4. (1) $a = -\frac{1}{2}, b = 1$, (2) $\frac{1}{16}$.

5. $a = 2, \frac{1}{4}, 2\sqrt{2} - \frac{9}{4}, 0$. 6. $\left(\frac{2}{3}\right)^3$.

2.2 节

1. (1) $\frac{5}{2}, \frac{49}{6}$, (2) $\frac{133}{6}$. 2. 0.301, 0.322. 3. 44.64. 4. 256.

5. $\frac{4}{3}$. 6. $\frac{1}{3}$.

2.3 节

1. $C_{20}^{18} 0.7^{18} 0.3^2 + C_{20}^{19} 0.7^{19} 0.3^1 + 0.7^{20}$. 2. $n = 20, p = 0.3$.

3. $EX = DX = 2$. 4. 1 或者 2. 5. $\frac{1}{2e}$.

6. $P(X = k) = \frac{1}{3} \left(\frac{2}{3} \right)^{k-1}$, $k=1, 2, \dots$. 7. 0.264.

2.4 节

1. $EY = \frac{11}{3}$, $DY = \frac{256}{45}$. 2. $\frac{20}{27}$. 3. $e^{-3.6} + 9e^{-1.2} - 6e^{-2.4} - 3$.

4. 0.102.

2.5 节

1. $\frac{Y}{p} \begin{array}{c|ccc} & 0 & 1 & 4 \\ \hline & 0.3 & 0.6 & 0.1 \end{array}$.

2. $\frac{Y}{p} \begin{array}{c|ccc} & -1 & 0 & 1 \\ \hline & 0.4^3 & 1.2 \times 0.6^2 & 1 - 0.4^3 - 1.2 \times 0.6^2 \end{array}$.

3. $f_Y(y) = \begin{cases} \frac{y-3}{8}, & 3 < y < 7 \\ 0, & \text{其它} \end{cases}$.

4. $f_Y(y) = \begin{cases} \frac{1}{4\sqrt{y}}, & 0 < y \leq 4 \\ 0, & \text{其它} \end{cases}$.

第二章 自测题

一. 1. $N(0,1)$. 2. $\frac{1}{3}, \frac{5}{9}$. 3. $\frac{1}{2}, \frac{1}{\pi}$. 4. 1. 5. $2 - 2F(a)$.

6. $\frac{1}{3} f_X(-\frac{y}{3})$. 7. $\frac{1}{3}$. 8. $\frac{X}{p} \begin{array}{c|ccc} & -1 & 0 & 2 \\ \hline & 0.4 & 0.4 & 0.2 \end{array}$. 9. $\frac{115}{132}$. 10. $\frac{1}{4}$.

11. $4, \frac{3}{4}, f(x) = \begin{cases} 8x^{-3}, & x > 2 \\ 0, & x \leq 2 \end{cases}$. 12. $2, e^{-200}$.

二. 1. (1) $\frac{\pi}{2}$, (2) $\frac{1}{2}$, (3) $F(x) = \begin{cases} 0, & x \leq 0 \\ 1 - \cos x, & 0 < x \leq \frac{\pi}{2} \\ 1, & x > \frac{\pi}{2} \end{cases}$.

2. (1) $f(x) = \begin{cases} \frac{2}{\pi} \cdot \frac{1}{1+x^2}, & -1 \leq x < 1 \\ 0, & \text{其它} \end{cases}$, (2) $\frac{2}{\pi}, \frac{4}{\pi} - 1$.

3. $\frac{1}{2} - \frac{3}{2}e^{-2}, \frac{3}{2}e^{-2} - \frac{23}{8}e^{-4} - \frac{1}{8}$.

4. $\frac{Y}{p} \begin{array}{c|cccc} & -3 & 1 & 5 & 9 \\ \hline & 0.2 & 0.1 & 0.3 & 0.4 \end{array}, \frac{Z}{p} \begin{array}{c|ccc} & 0 & 1 & 4 \\ \hline & 0.1 & 0.5 & 0.4 \end{array}$.

5. $f_Y(y) = \begin{cases} \frac{1}{\sqrt{2\pi}y} e^{-\frac{(\ln y)^2}{2}}, & y > 0 \\ 0, & y \leq 0 \end{cases}$.

三. 1. $\frac{X}{p} \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline & \frac{1}{35} & \frac{12}{35} & \frac{18}{35} & \frac{3}{35} \end{array}, \frac{22}{35}$.

2. $5e^{-900} - 2$.

3. (1) $5(3e^{-2})(1-3e^{-2})^4$, (2) $1-(1-3e^{-2})^5$.

4. $3 \times 0.09757^2 \times (1-0.09757)$.

第三章 3.1 节

1.

$\begin{matrix} Y \\ X \end{matrix}$	1	2	3
0	3/64	18/64	6/64
1	0	9/64	18/64
2	0	9/64	0
3	1/64	0	0

2. (1) $a=0.1$ 。

(2)

X	0	1
P	0.5	0.5

Y	0	1	2
P	0.4	0.3	0.3

(3) 0.5. (4) 0.8. (5) 0.3.

3. (1)

$\begin{matrix} Y \\ X \end{matrix}$	1	2	3	4	5	6
1	1/36	1/36	1/36	1/36	1/36	1/36
2	0	2/36	1/36	1/36	1/36	1/36
3	0	0	3/36	1/36	1/36	1/36
4	0	0	0	4/36	1/36	1/36
5	0	0	0	0	5/36	1/36
6	0	0	0	0	0	6/36

(2)

X	1	2	3	4	5	6
P	6/36	6/36	6/36	6/36	6/36	6/36
Y	1	2	3	4	5	6
P	1/36	3/36	5/36	7/36	9/36	11/36

(3) 21/36. (4) 8/36.

4. (1) $f(x, y) = \begin{cases} 1/2 & 0 \leq x \leq 2, 0 \leq y \leq 1 \\ 0 & \text{其他} \end{cases}$; (2) $f(x) = \begin{cases} 1/2 & 0 \leq x \leq 2 \\ 0 & \text{其他} \end{cases}$,

$f(y) = \begin{cases} 1 & 0 \leq y \leq 1 \\ 0 & \text{其他} \end{cases}$; (3) 2/3.

5. (1) 1/3. (2) 5/12.

$$(3) f(x) = \begin{cases} 2x^2 + \frac{2}{3}x & 0 \leq x \leq 1 \\ 0 & \text{其他} \end{cases}, f(y) = \begin{cases} \frac{1}{3} + \frac{1}{6}y & 0 \leq y \leq 2 \\ 0 & \text{其他} \end{cases}.$$

$$(5) P(X \geq 2 | Y = 4) = 1 - F(2 | 4) = 1 - \int_{-\infty}^2 \frac{f(x, 4)}{f(4)} dx = \frac{1}{2}.$$

3.3 节

1. (1)

$$6. (1) 15. (2) f(x) = \begin{cases} 5x^4 & 0 \leq x \leq 1 \\ 0 & \text{其他} \end{cases},$$

$$f(y) = \begin{cases} 15y^2(\frac{1}{2} - \frac{1}{2}y^2) & 0 \leq y \leq 1 \\ 0 & \text{其他} \end{cases}. (3) 1/243.$$

3.2 节

$$1. P(X_1 = 0 | X_2 = 1) = 1/3, P(X_1 = 1 | X_2 = 1) = 2/3.$$

2. 不独立. 3. 6, 独立.

$$4. f(x) = \begin{cases} \frac{21}{4}(e^{-3x} - e^{-7x}) & x \geq 0 \\ 0 & x < 0 \end{cases}, f(y) = \begin{cases} 7e^{-7y} & y \geq 0 \\ 0 & y < 0 \end{cases}. \text{不独立.}$$

$$5. (1) f(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & x \leq 0 \end{cases}, f(y) = \begin{cases} ye^{-y} & y > 0 \\ 0 & y \leq 0 \end{cases}.$$

(2) X, Y 不独立.

$$(3) \text{当 } y > 0 \text{ 时, } f_{X|Y}(x|y) = \frac{f(x, y)}{f(y)} = \begin{cases} \frac{1}{y} & 0 < x < y \\ 0 & \text{其他} \end{cases}.$$

$$(4) P(X + 2Y \leq 1) = \int_0^{\frac{1}{3}} dx \int_x^{\frac{1-x}{2}} e^{-y} dy = 1 + 2e^{-\frac{1}{2}} - 3e^{-\frac{1}{3}}.$$

X \ Y	1	2
	0	1
0	1/10	3/20
1	3/10	9/20

(2)

X+Y	1	2	3	XY	0	1	2
P	1/10	9/20	9/20	P	1/4	3/10	9/20

$$2. f(z) = \begin{cases} \frac{1}{2}(\ln 2 - \ln z) & 0 < z < 2 \\ 0 & \text{其他} \end{cases}. 3. 3/4, 8/5, 6/5, 47/20.$$

$$4. 5/3. 5. 4/3, 5/8, 47/24, 5/6, 5/8.$$

3.4 节

1. (1) 0, 0. (2) 不独立, 不相关. 2. 4. 3. (1) 27, (2) 6.

$$4. EX = \frac{7}{6}, EY = \frac{7}{6}, EX^2 = EY^2 = \frac{5}{3}, DX = DY = \frac{11}{36}.$$

$$EXY = \frac{4}{3}, COV(X, Y) = -\frac{1}{36}, \rho_{XY} = -\frac{1}{11}, D(X - Y) = \frac{6}{9}.$$

$$5. 4/5, 3/5, 2/75, 1/25, 1/50, \sqrt{6}/4.$$

3.5 节

1. 0.02275. 2. 0.90147. 3. 0.00003; 40 万元. 4. $m=233958$.

2. (1)

X	0	1
P	0.65	0.35

第三章 自测题

- 一. 1. $a+b=1/3$, $a=2/9$, $b=1/9$. 2. $1/4$, $1/8$. 3. $\frac{1}{3}$.

4. $f_{X|Y}(x|y) = \begin{cases} \frac{2x}{y^2} & 0 \leq x \leq y \leq 1 \\ 0 & \text{其他} \end{cases}$. 5. 16.59. 6. 97, 97.

Y	-1	0	1
P	0.25	0.35	0.4

(2) -0.1025, 1.06, -0.08.

7. $X - 2Y + 11 \sim N(4, 17)$.

3. (1) (X, Y) 的概率分布为:

- 二. 1. B. 2. C. 3. A. 4. B. 5. B. 6. C. 7. B.

三. 1.

X \ Y	0	1
0	$\frac{2}{3}$	$\frac{1}{12}$
1	$\frac{1}{6}$	$\frac{1}{12}$

Y	2	3	4	Pi.
X				
1	1/6	1/6	1/6	3/6
2	0	1/6	1/6	2/6
3	0	0	1/6	
p.j	1/6	2/6	3/6	

(2) $\rho_{XY} = \frac{\text{Cov}(X, Y)}{\sqrt{DX} \cdot \sqrt{DY}} = \frac{\sqrt{15}}{15}$.

(3) Z 的概率分布为:

Z	0	1	2
P	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{1}{12}$

- $5/3$, $10/3$, $5/9$, $5/9$.

4. (1) 随机变量 X 和 Y 的联合概率密度为

$$f(x, y) = \begin{cases} \frac{1}{x}, & 0 < y < x < 1, \\ 0, & \text{其他.} \end{cases}$$

$$(2) f_Y(y) = \begin{cases} -\ln y, & 0 < y < 1, \\ 0, & \text{其他.} \end{cases} \quad (3) 1 - \ln 2.$$

$$5. (1) f(x) = \begin{cases} 1 + 2x - 3x^2 & 0 \leq x \leq 1 \\ 0 & \text{其他} \end{cases}, f(y) = \begin{cases} 3y^2 & 0 \leq y \leq 1 \\ 0 & \text{其他} \end{cases}, \text{不独立.}$$

$$(2) 1/3. \quad (3) 1/3.$$

$$6. a = 0.086.$$

第四章 4.1、4.2 节

$$1. \bar{X} = 7, S^2 = 1.5.$$

$$2. (1) p, \frac{pq}{n}, (2) np, pq, (3) \lambda, \frac{\lambda}{n}, (4) \frac{a+b}{2}, \frac{(b-a)^2}{12n},$$

$$(5) \frac{1}{\lambda}, \frac{1}{n\lambda^2}.$$

$$3. \mu, \frac{\sigma^2}{n}, \sigma^2. \quad 4. (1) \frac{\lambda^{\sum_{i=1}^n x_i}}{x_1! \times \cdots \times x_n!} e^{-n\lambda}, (2) \lambda^n e^{-\lambda \sum_{i=1}^n x_i}.$$

4.3、4.4 节

$$1. \Phi(1.6667) + \Phi(1.1111) - 1. \quad 2. \alpha = \frac{1}{20}, \beta = \frac{1}{100}.$$

$$3. 0.025, 0.01. \quad 4. 16. \quad 6. \frac{1}{8}. \quad 7. F(7, 9).$$

第四章 自测题

$$\text{一. } 1. C. \quad 2. B. \quad 3. A. \quad 4. A. \quad 5. B. \quad 6. C. \quad 7. D. \quad 8. D. \quad 9. D. \quad 10. B. \\ 11. C. \quad 12. AC. \quad 13. B.$$

$$\text{二. } 1. 1, \frac{9}{n}. \quad 2. 115.6, 13427.66. \quad 3. n, \frac{n}{2}. \quad 4. t(2).$$

$$5. F(2n, n). \quad 6. b(n, p), N(p, \frac{pq}{n}). \quad 7. N(0, \frac{9\sigma^2}{20}). \quad 8. \frac{\sqrt{6}}{2}.$$

$$\text{三. } 1. 16. \quad 2. 2 - 2\Phi(\frac{0.3}{\sqrt{0.5}}).$$

$$3. a = \frac{1}{8}, b = \frac{1}{12}, c = \frac{1}{16}, U \sim \chi^2(3).$$

第五章 5.1 节

$$1. (1) \text{是统计量, 不是无偏的}; (2) \text{不是统计量}; \\ (3) \text{是无偏统计量}; (4) \text{是统计量, 不是无偏的}.$$

$$2. a = \frac{1}{2}. \quad 4. \hat{\mu}_2 \text{最有效}.$$

5.2 节

$$1. (1) \hat{\alpha} = \frac{2\bar{X} - 1}{1 - \bar{X}}; \quad \hat{\alpha}_L = -\frac{n}{\sum_{i=1}^n \ln X_i} - 1.$$

$$(2) \hat{\theta} = \frac{1}{\bar{X}}; \hat{\theta}_L = \frac{1}{\bar{X}}.$$

$$(3) \hat{\lambda} = \bar{X}; \hat{\lambda}_L = \bar{X}.$$

$$2. \frac{5}{6}, \frac{5}{6}.$$

5.3 节

$$1. (11.366, 14.634).$$

$$2. (1) (2.121, 2.129);$$

$$(2) (1.668, 2.582).$$

$$3. (1) (71.852, 81.348);$$

$$(2) (59.478, 219.374).$$

5.4、5.5 节

$$1. |u| \approx 1.23 < 1.96, \text{ 接受 } H_0.$$

$$2. |u| \approx 3.33 > 1.96, \text{ 拒绝 } H_0.$$

$$3. |t| \approx 0.923 < t_{0.01}(9) = 2.821, \text{ 接受 } H_0.$$

$$4. |t| \approx 1.995 < t_{0.025}(5) = 2.571, \text{ 接受 } H_0.$$

$$5. t \approx 0.136 < t_{0.05}(8) = 1.86, \text{ 接受 } H_0.$$

$$6. t \approx 2.788 > t_{0.05}(9) = 1.833, \text{ 拒绝 } H_0.$$

$$7. \chi_0^2 \approx 1.5278, \chi_{0.025}^2(4) = 11.143, \chi_{0.975}^2(4) = 0.484.$$

$$0.484 < 1.5278 < 11.143, \text{ 接受 } H_0.$$

$$8. \chi_0^2 \approx 17.858, \chi_{0.025}^2(4) = 11.143, \chi_{0.975}^2(4) = 0.484.$$

$$11.858 > 11.143, \text{ 拒绝 } H_0.$$

$$9. \chi_0^2 \approx 9.929, \chi_{0.05}^2(7) = 14.067.$$

$$9.929 < 14.067, \text{ 接受 } H_0.$$

$$10. \chi_0^2 \approx 15.68, \chi_{0.05}^2(8) = 15.507.$$

$$15.68 > 15.507, \text{ 拒绝 } H_0.$$

$$11. (1) |t| \approx 0.917 < t_{0.025}(24) = 2.064, \text{ 接受 } H_0.$$

$$(2) \chi_0^2 \approx 34.66 < \chi_{0.05}^2(24) = 36.415 \text{ 接受 } H_0.$$

满足要求.

5.6 节

$$1. |u| = 2.5 > u_{\frac{\alpha}{2}} = 1.96, \text{ 拒绝 } H_0.$$

$$2. u = 3.947 > u_{0.05} = 1.64, \text{ 拒绝 } H_0.$$

$$3. |t| \approx 0.2648 < t_{0.025}(13) = 2.16, \text{ 接受 } H_0.$$

$$4. F = 1.1724, F_{0.05}(15, 12) = 2.62, F_{0.95}(15, 12) = 0.4032,$$

接受 H_0 .

5. $F \approx 3.673 > F_{0.05}(7,9) = 3.29$, 拒绝 H_0 .

2. (1) $|t| \approx 0.209 < t_{0.025}(9) = 2.262$, 接受 H_0 .

6. (1) $F \approx 1.552, F_{0.025}(20,20) = 2.464, F_{0.975}(20,20) = 0.406$, 接受总体方差相等.

(2) $\chi_0^2 \approx 36.552 > \chi_{0.05}^2(9) = 16.919$, 拒绝 H_0 , 机器工作不正常.

(2) $|t| \approx 2.849 > t_{0.025}(40) = 2.021$, 拒绝 H_0 .

3. (1) $F = 1, F_{0.025}(26,25) = 2.219, F_{0.975}(26,25) = 0.453$, 接受总体方差相等.

(2) $|t| \approx 0.262 < t_{0.025}(51) = 2.008$, 接受 H_0 .

第五章 自测题

一. 1. $\bar{X}, \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2$. 2. $\bar{X}$.

3. $\alpha^n (\prod_{i=1}^n x_i)^{\alpha-1}$. 4. $\frac{1}{4}, \frac{7}{8}$. 5. $1-\alpha$.

6. $H_0: \mu \leq 14, H_1: \mu > 14$.

7. 小概率原理.

8. $C = \left\{ (x_1, x_2, \dots, x_n) : \left| \frac{\bar{x} - 10}{s/\sqrt{n}} \right| > 2.26 \right\}$.

二. 1. $\checkmark$ 2. $\times$ 3. $\times$ 4. $\checkmark$ 5. $\times$ 6. $\times$

三. 1. 均是, $\hat{\mu}_2$ 最有效. 2. $\hat{p}_L = \frac{1}{\bar{X}}$. 3. $\hat{\sigma}_L = \frac{1}{n} \sum_{i=1}^n X_i$.

4. (14.41, 14.49). 5. (4.243, 24.372).

四. 1. (1) (30.14, 33.86), (2) $u = 2 > u_{0.05} = 1.64$, 拒绝 H_0 .